Exact Boundary Conditions for the Teukolsky Equation

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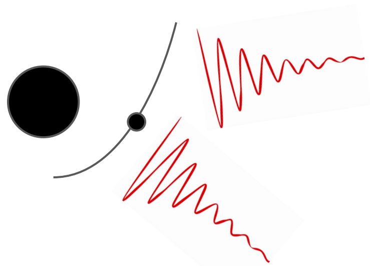
Motivation: EMRI waveforms

- LISA is expected to detect EMRI signals modeled by Black hole perturbation theory.



- The Teukolsky master equation describes scalar, vector, and tensor field perturbations in the space-time of Kerr black holes and given by

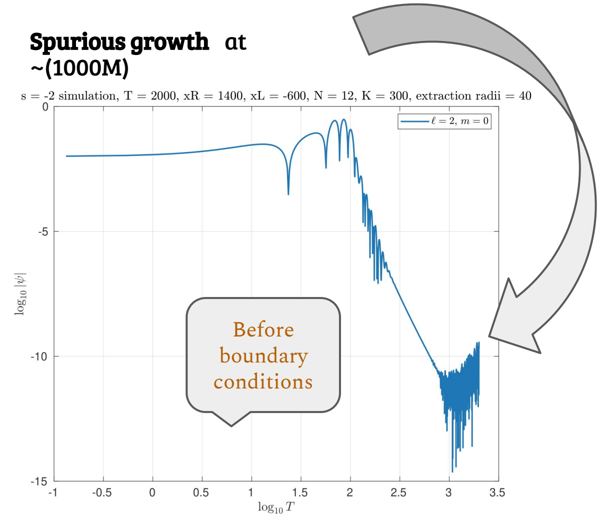
$$-\left[\frac{(r^2+a^2)^2}{\Delta} - a^2\sin^2\theta\right]\partial_{tt}\Psi - \frac{4Mar}{\Delta}\partial_{t\phi}\Psi$$
$$-2s\left[r - \frac{M(r^2-a^2)}{\Delta} + ia\cos\theta\right]\partial_t\Psi$$
$$+\Delta^{-s}\partial_r\left(\Delta^{s+1}\partial_r\Psi\right) + \frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\,\partial_\theta\Psi\right) +$$
$$\left[\frac{1}{\sin^2\theta} - \frac{a^2}{\Delta}\right]\partial_{\phi\phi}\Psi + 2s\left[\frac{a(r-M)}{\Delta} + \frac{i\cos\theta}{\sin^2\theta}\right]\partial_\phi\Psi$$
$$-\left(s^2\cot^2\theta - s\right)\Psi = -4\pi\left(r^2+a^2\cos^2\theta\right)T,$$

$$\Delta = r^2 - 2Mr + a^2$$

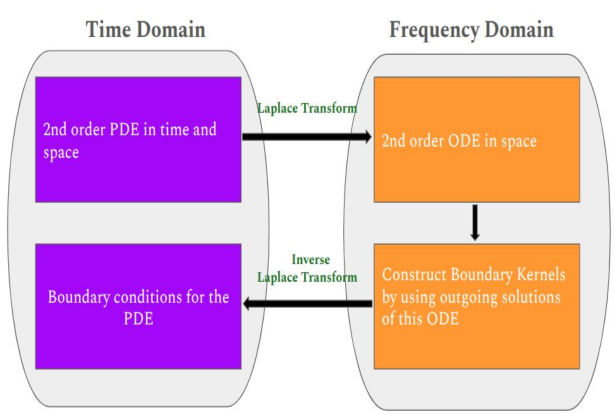
- s is the spin weight of the field, where the s = ±2 versions of these equations describe gravitational waves, and T is a source term.

- We solve the Teukolsky equation using **discontinuous Galerkin** methods.

Problems in Boyer-Lindquist coordinates ?

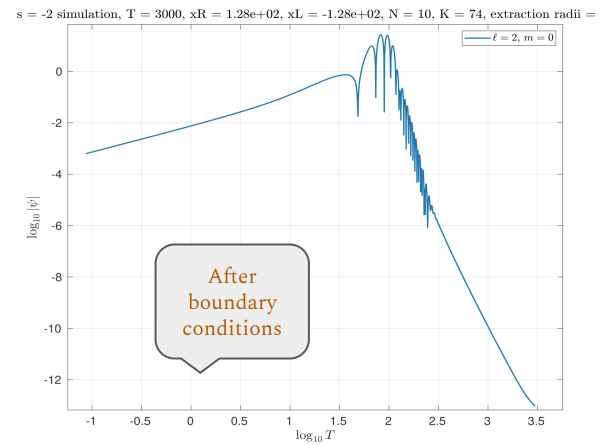


Outer Boundary conditions using Boundary Kernels



Exact BCs resolve Blow up

Simulation with exact boundary conditions applied: ✓



Validation using power-law decay of tails: ✓